Lessons From Soft-Collinear Effective Theory

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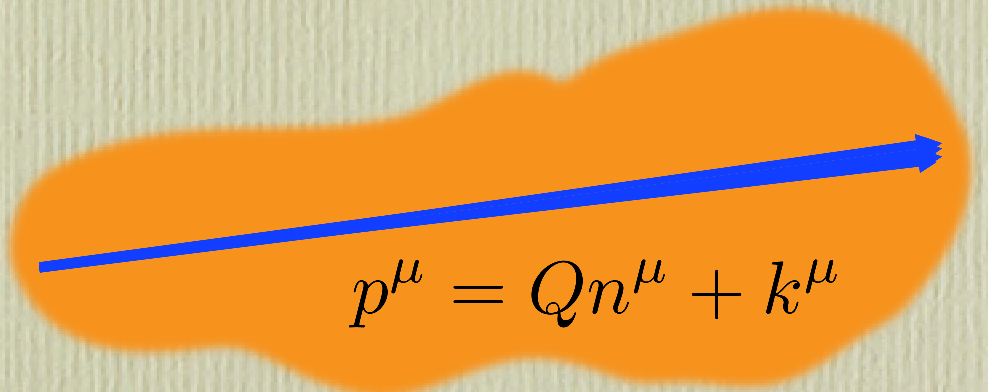
Outline

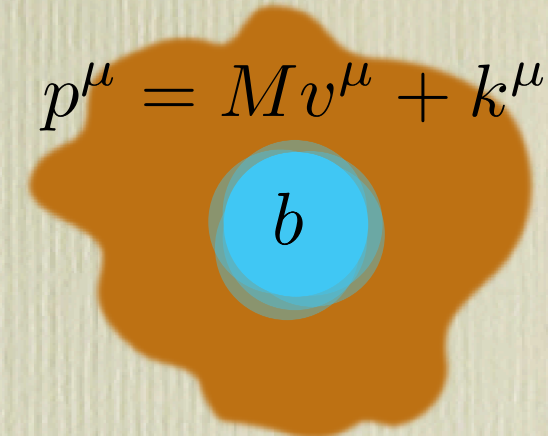
- Overview of Soft-Collinear Effective Theory
    - Motivation and purpose
- Introduction to SCET through an example: $B \to X_s \gamma$
- SCET Lagrangian and properties
- Some more applications
    - Deep inelastic scattering
    - J/ψ production in e^+e^- annihilation at $\sqrt{s} = 10.6\,\mathrm{GeV}$

Soft-Collinear Effective Theory: an Overview

- The basic idea is to understand an approximately massless highly energetic particle interacting with a soft background



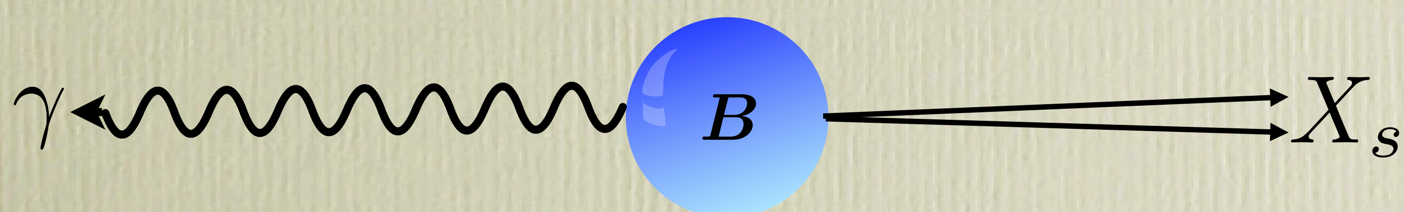
Analogy with HQET



$$k \sim \Lambda_{\rm QCD} \ll Q$$

- We first introduced the theory in the context of the decay rate for $B \to X_s \gamma$ when the final state decay product X_s has energy of order M_B and invariant mass $\ll M_B$

(C. Bauer, SF , M. Luke, Phys. Rev. D 63: 014006, 2001)



Soft-Collinear Effective Theory: an Overview

- SCET is a framework for understanding:
  - Factorization
  - Summation of logs that arise at the edges of phase space
  - Systematic method for including power corrections
- SCET Lagrangian, symmetries, and properties
  - **C. Bauer, SF, D. Pirjol, I. Stewart, Phys. Rev. D63: 114020, 2001**
- Followed by important papers on
  - Gauge symmetries and factorization properties

    C. Bauer, I. Stewart, Phys. Lett. B516: 134, 2001
    C.Bauer, D. Pirjol, I. Stewart, Phys. Rev. D65: 054022, 2002
    C. Bauer, SF, D. Pirjol, I. Rothstein, I. Stewart, Phys. Rev. D66: 014017, 2002
  - Subleading contributions

    J. Chay, C. Kim, Phys. Rev. D65, 114016, 2002
    M. Beneke, A.P. Chapovsky, M. Diehl, T. Feldmann, Nucl. Phys. B643 , 2002
  - More ...

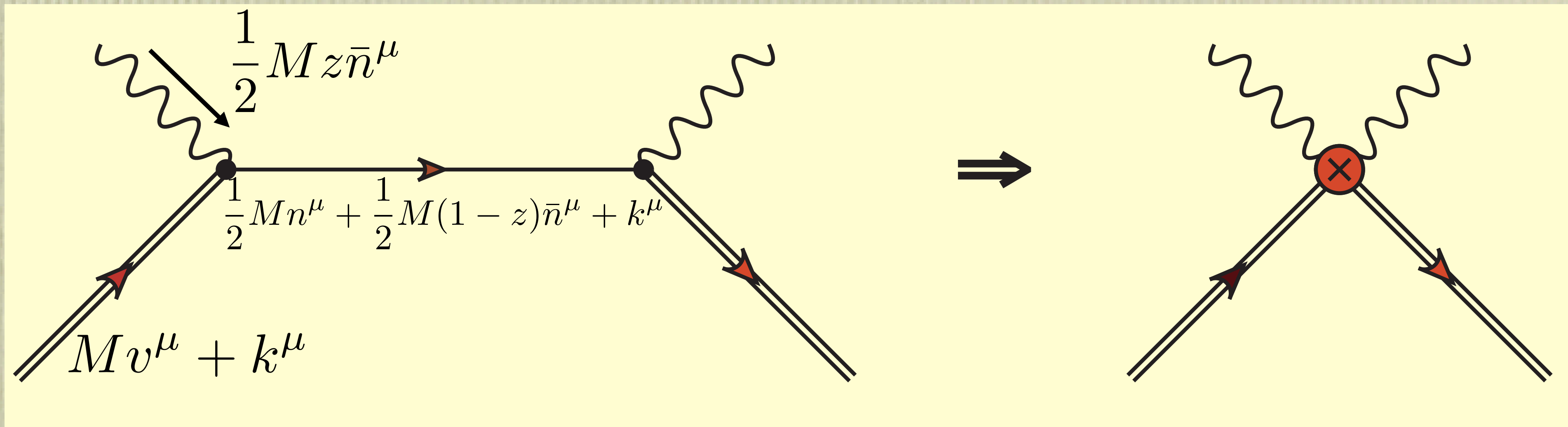
Return to the beginning: $B \to X_s\gamma$

(C. Bauer, S. F. , M. Luke, Phys. Rev. D 63: 014006, 2001)

- OPE in inverse heavy quark mass for inclusive observables
  - Experimental cuts restrict phase space
- Semi-inclusive observables: a set of subleading contributions in the OPE are enhanced
  - Sum enhanced operators into a non-perturbative shape function
- Sudakov Logarithms of the form $\alpha_s \log^2 \Delta + \alpha_s \log \Delta + \ldots$ arise for a region of order Δ around the maximal photon energy
  - Logarithms summed using perturbative QCD techniques

Return to the beginning: $B \to X_s\gamma$

What is going on?



$$z = 2E_\gamma/M \qquad \bar{n}^\mu = (1,0,0,1) \qquad n^\mu = (1,0,0,-1)$$

$$P_{X_s}^2 = M^2(1-z) + Mn\cdot k + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2)$$

- $P_X^2 \approx M^2(1-z) \gg \Lambda_{\mathrm{QCD}}$:OPE converges
- $M(1-z) \sim \Lambda_{\mathrm{QCD}}$ then $P_X^2 \approx M^2(1-z) + Mn\cdot k \gg \Lambda_{\mathrm{QCD}}$: twist expansion
- $P_X^2 \approx \Lambda_{\mathrm{QCD}}$:Resonance region

Beyond $1/M$

Resumming terms of the form $\frac{n\cdot k}{M(1-z)}$ from the expansion of the propagator results in a shape function

M. Neubert, Phys. Rev D49, 3392, 1992
I. Bigi *et al.* Int. J. Mod. Phys. A9, 2467, 1994

Decay rate is a convolution:

$$\frac{d\Gamma}{dz} = \int d\xi \; f(\xi) \frac{d\Gamma_p}{dz}(\xi - z)$$

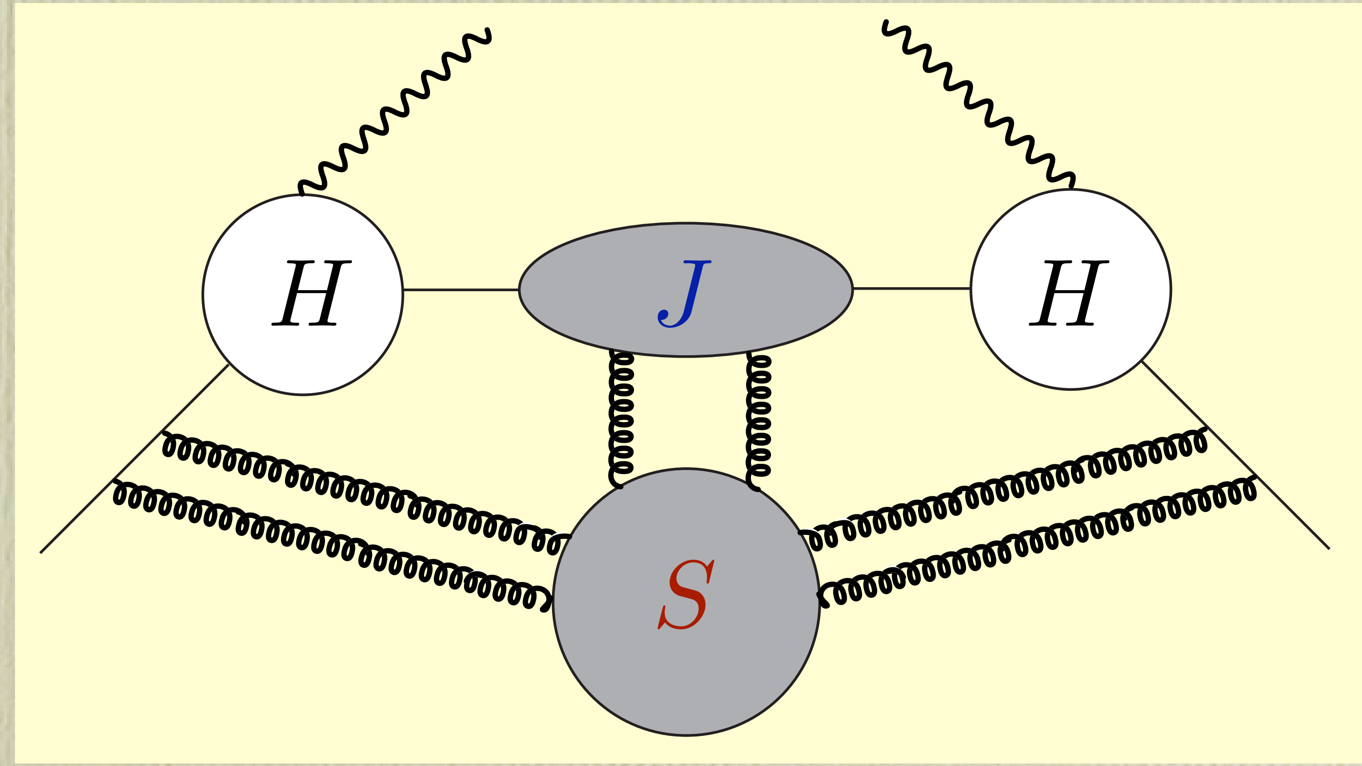
- $f(\xi)$ is the matrix element of a non-local operator
- Measures residual momentum of b-quark in the n direction
- Universal

Goal 1: Understand the origin of the non-local operator which gives the shape function in an EFT framework

Peturbative Summation

- Logs are summed using perturbative factorization techniques

G. P. Korchemsky and G. Sterman Phys. Lett. B340, 96 (1994);
R. Akhoury and I. Z. Rothstein, Phys. Rev. D54, 2349 (1996).



- In moment space: $M_N = \int dz\, z^{N-1} \frac{d\Gamma}{dz} = S_N J_N H$
  - Sum logs of the form: $\alpha_s \log^2 N + \alpha_s \log N$

Goal 2: Sum these logs in an EFT using the RGEs

A New Degree of Freedom

- The final state is almost light-like

$$P^{\mu}_{X_s} = \frac{1}{2} M n^{\mu} + \frac{1}{2} M (1 - z) \bar{n}^{\mu} + k^{\mu}$$

$$\sqrt{P^2_{X_s}} \approx M \sqrt{\frac{\Lambda_{\mathrm{QCD}}}{M}} \ll M$$

- Include these collinear modes in our EFT
  - Momentum scaling $\lambda \sim \sqrt{\frac{\Lambda_{\mathrm{QCD}}}{M}}$

$$p = (p^{+}, p^{-}, \vec{p}_{\perp}) = (n \cdot p, \bar{n} \cdot p, \vec{p}_{\perp}) \sim M(\lambda^2, 1, \lambda)$$

- In addition to soft modes

$$k \sim (\Lambda_{\mathrm{QCD}}, \Lambda_{\mathrm{QCD}}, \Lambda_{\mathrm{QCD}}) \sim M(\lambda^2, \lambda^2, \lambda^2)$$

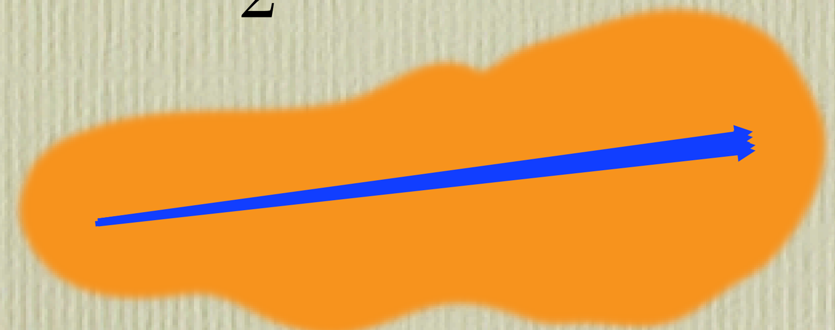
Couple these two modes in the new EFT

Soft-Collinear Effective Theory

C. Bauer, S F , M. Luke, Phys. Rev. D 63: 014006, 2001
C. Bauer, SF, D. Pirjol, I. Stewart, Phys. Rev. D63: 114020, 2001
C. Bauer, I. Stewart, Phys. Lett. B516: 134, 2001
C.Bauer, D. Pirjol, I. Stewart, Phys. Rev. D65: 054022, 2002

SCET

$$p^\mu = \frac{1}{2}\bar{n}\cdot p\, n^\mu + p_\perp^\mu + k^\mu$$

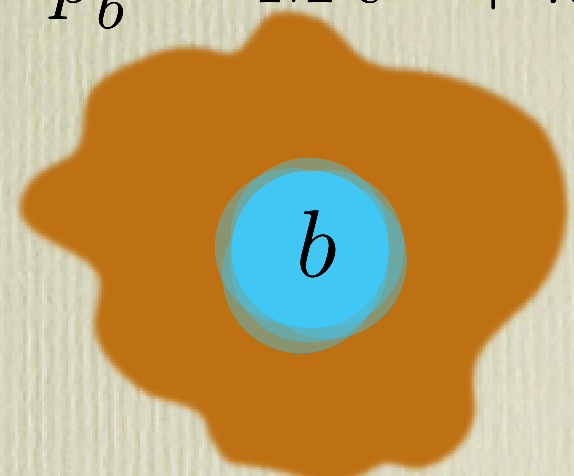


$$k \sim \Lambda_{\rm QCD} \ll p_\perp \ll \bar{n}\cdot p$$

Analogy with

HQET

$$p_b^\mu = M v^\mu + k^\mu$$



- Effective theory of an approximately massless particle interacting with a soft background

Soft-Collinear Effective Theory

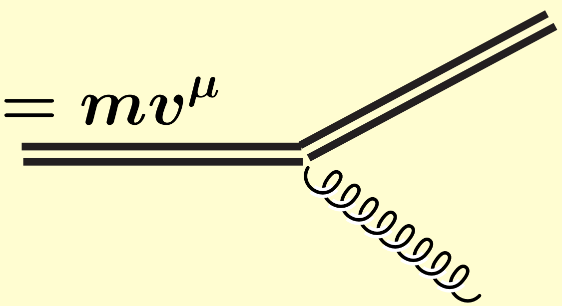
- Analogy with HQET breaks down:

HQET

Not Allowed!!!

$p'^{\mu} = mzv^{\mu}$

$p^{\mu} = mv^{\mu}$



$q^{\mu} = m(1-z)v^{\mu}$

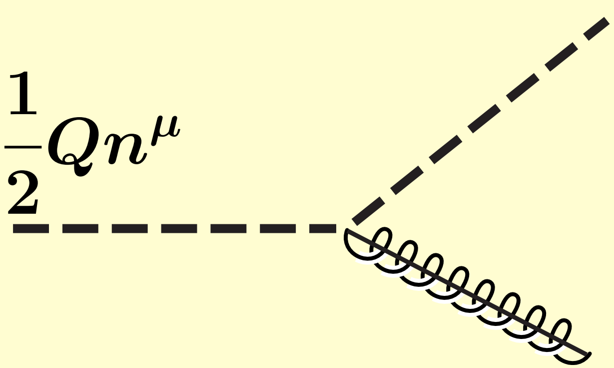
$q^2 = m^2(1-z)^2$

SCET

O.K.

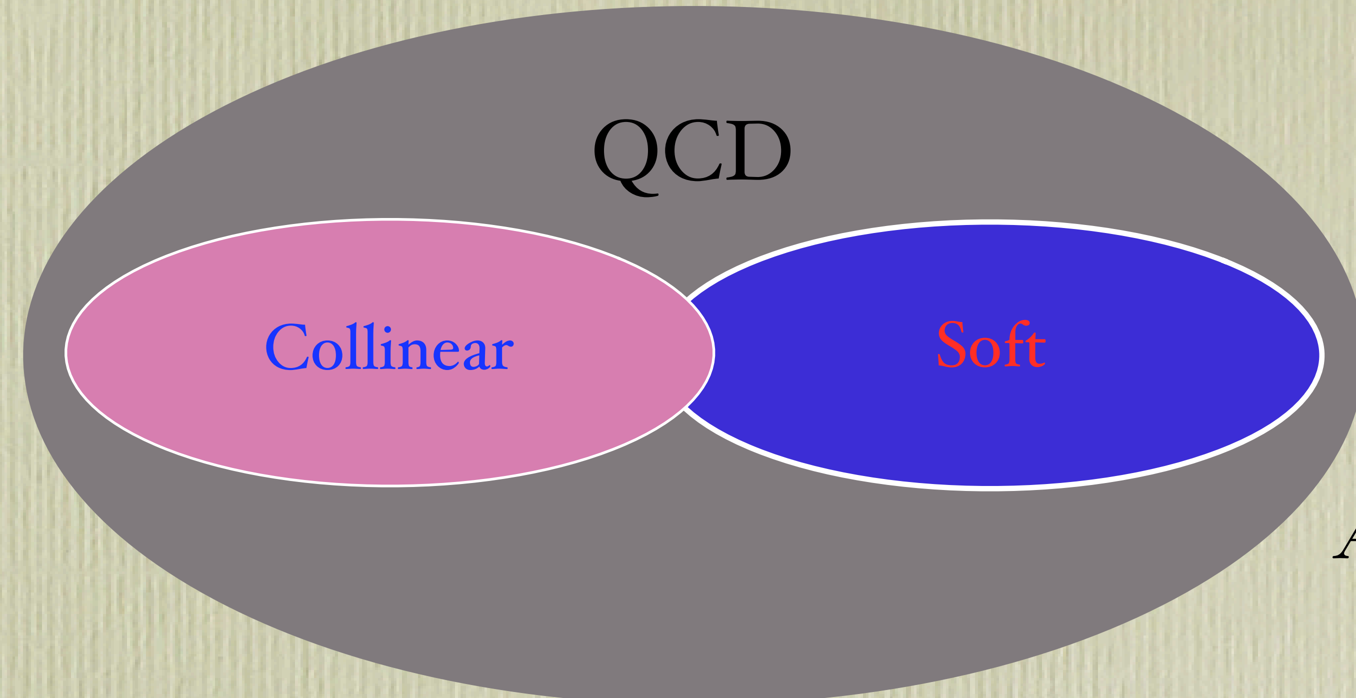
$p'^{\mu} = \frac{1}{2}zQn^{\mu}$

$p^{\mu} = \frac{1}{2}Qn^{\mu}$



$q^{\mu} = \frac{1}{2}(1-z)Qn^{\mu}$ $\quad q^2 = 0$

SCET Lagrangian



$$\psi(x) \to \psi_s(x) + \xi_n(x)$$

$$A^\mu(x) \to A^\mu_s(x) + A^\mu_n(x)$$

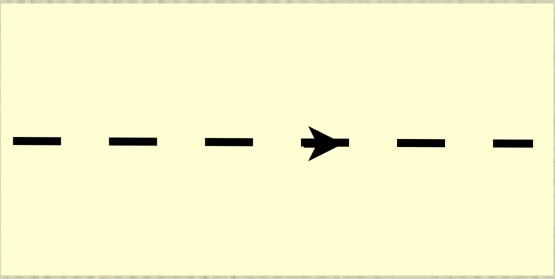
$$\mathcal{L}_c = \bar{\xi}_n \left\{ in \cdot D_c + i\not{D}_c^\perp \frac{1}{i\bar{n} \cdot D_c} i\not{D}_c^\perp + gn \cdot A_s \right\} \frac{\not{\bar{n}}}{2} \xi_n$$

$$\mathcal{L}_s = \bar{\psi}_s \, i\not{D}_s \psi_s$$

- Collinear sector: QCD in boosted frame
- Soft sector: QCD
- Coupled through a single term

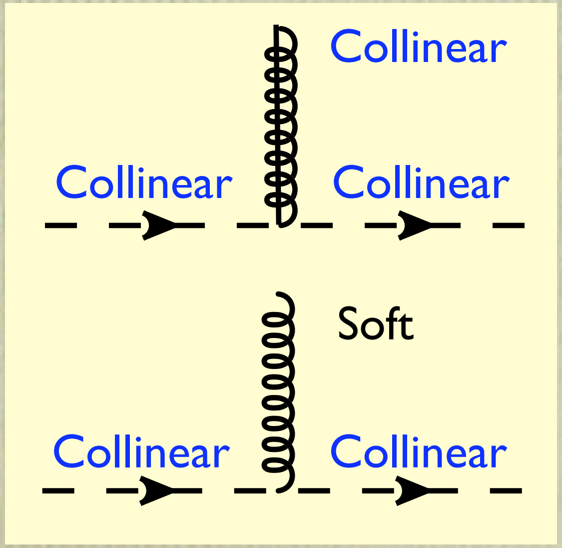
Feynman Rules

- Collinear Propagator



$$i\frac{\not n}{2}\frac{\bar n\cdot p}{n\cdot p\,\bar n\cdot p + p_\perp^2 + i\epsilon}$$

- Verticies



$$igT^A\left[n^\mu + \frac{\gamma_\mu^\perp \not p_\perp}{\bar n\cdot p} + \frac{\not p'_\perp\gamma_\mu^\perp}{\bar n\cdot p'} - \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n^\mu\right]\frac{\not{\bar n}}{2}$$

$$igT^A n^\mu \frac{\not{\bar n}}{2}$$

Symmetries

- Separate collinear and soft gauge symmetries
  - Powerful restriction on the form of operators allowed
  - Soft fields act as a background field to collinear fields
  - Any gauge symmetry connecting soft to collinear introduces a large scale
- Global $U(1)$ helicity spin symmetry
- Reparameterization invariance which is a consequence of Lorentz invariance of QCD
  - Relates operators

Currents

Two important points:

1. Introduce a collinear Wilson line: $W = \mathrm{P}\exp\left(ig\int_{-\infty}^{x} ds\;\bar{n}\!\cdot\! A_n(s\bar{n})\right)$

$$W^\dagger \xi_n(x) \to W^\dagger \xi_n(x)$$ under collinear gauge transformations

2. Wilson coefficients are a function of the large light-cone component of the collinear momentum

$$C(\bar{n}\!\cdot\! P)$$

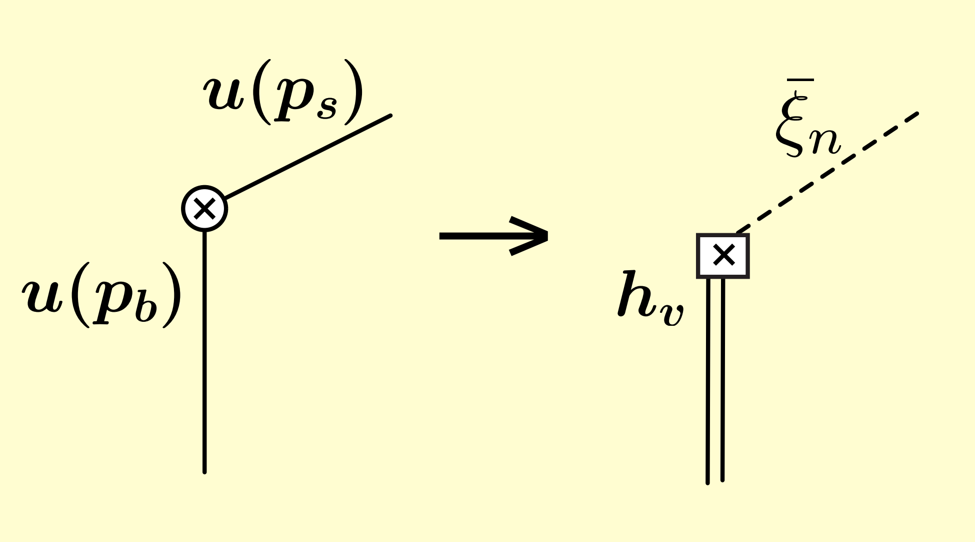
- Example: Heavy-light current at leading order in λ

$$\bar{u}(p_s)\Gamma u(p_b) \to \sum_i \bar{\xi}_n W C_i(\bar{n}\cdot \overleftarrow{P}_{\mathrm{op}})\Gamma_i h_v$$

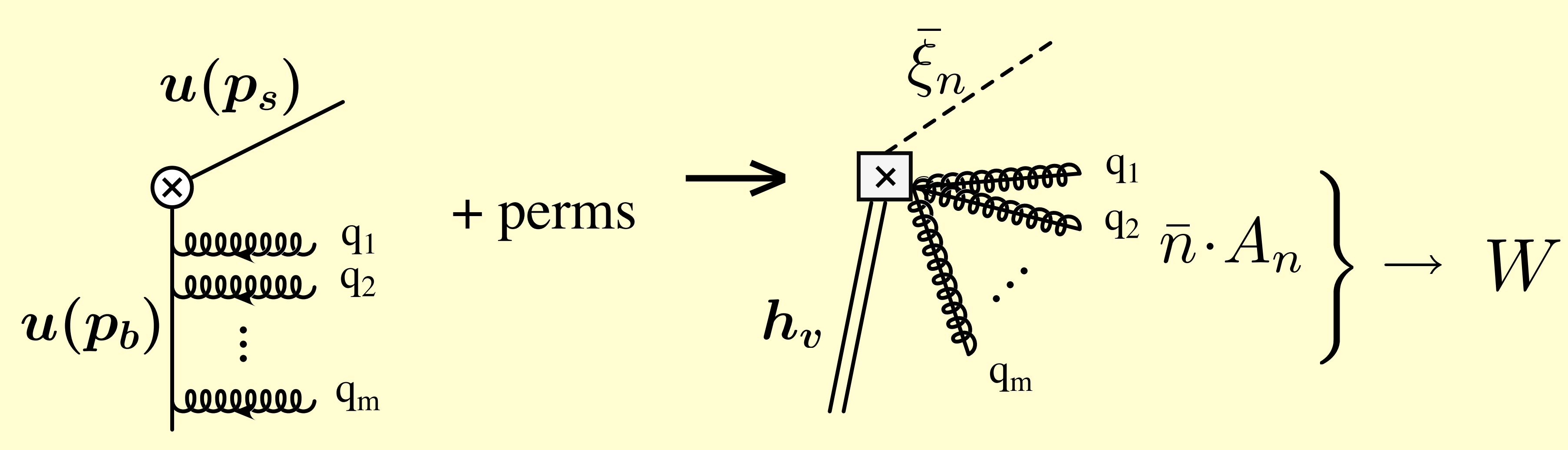
Heavy-Light Current

Origin of the collinear Wilson line

- Leading order in α_s



- Higher orders



Decoupling Collinear & Soft

- Decouple Soft from Collinear in the Lagrangian

1) Soft Wilson Line $$Y(x) = \mathrm{P}\exp\left(ig \int_{-\infty}^{x} ds\, n \cdot A_s(ns) \right)$$

2) Field Redefinition $$\xi_n(x) = Y(x)\xi_n^{(0)}(x)$$

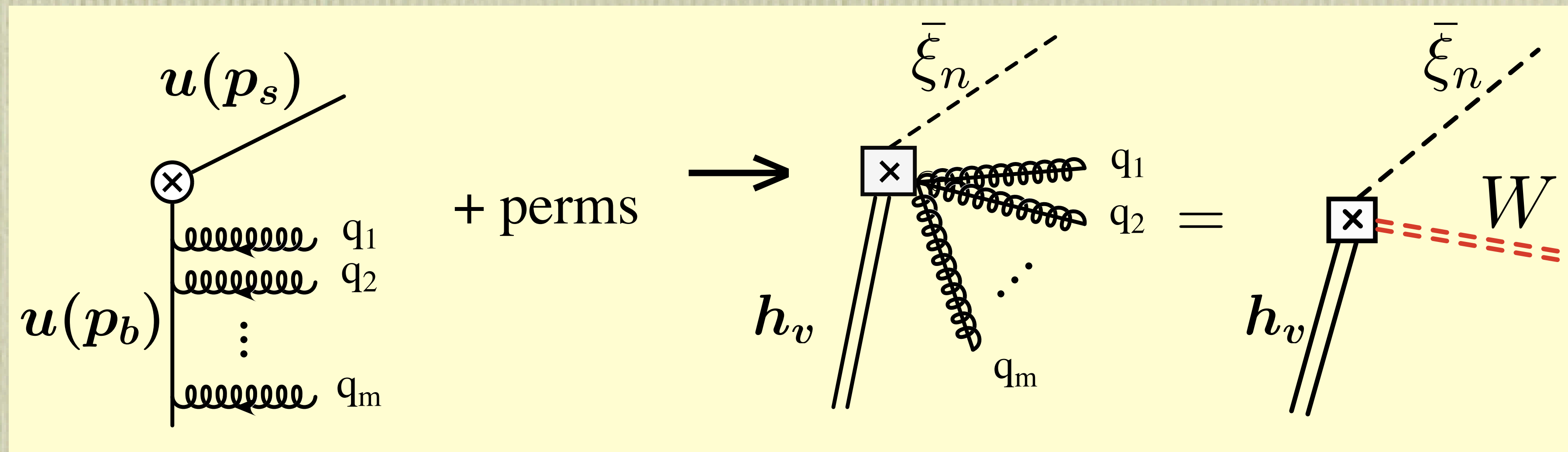
$$\mathcal{L}_c \to \bar{\xi}_n \left\{ in \cdot D_c + i\not{D}_c^{\perp} \frac{1}{i\bar{n} \cdot D_c} i\not{D}_c^{\perp} \right\} \frac{\bar{\not{n}}}{2} \xi_n$$

- Complicates vertex

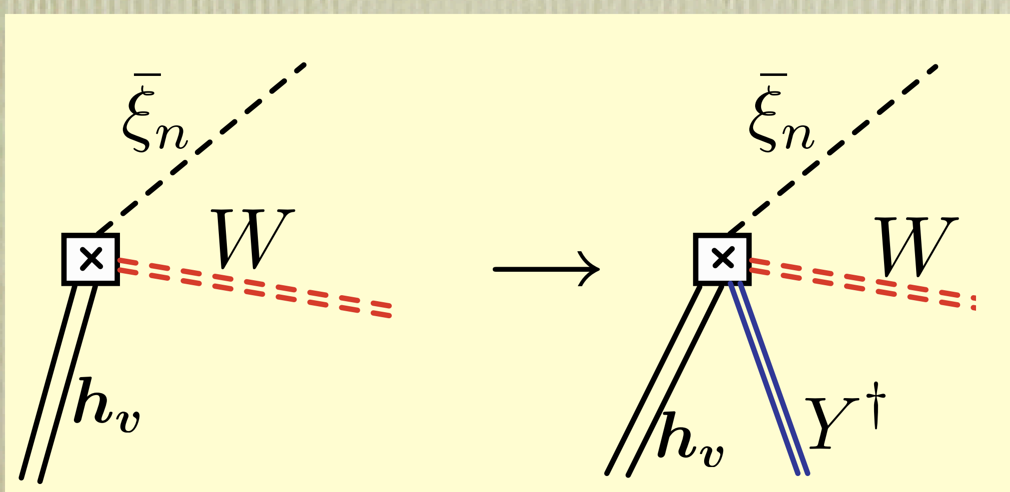
$$\bar{\xi}_n W \Gamma h_v \to \bar{\xi}_n^{(0)} W^{(0)} \Gamma Y^{\dagger} h_v$$

$B \to X_s\gamma$ in SCET

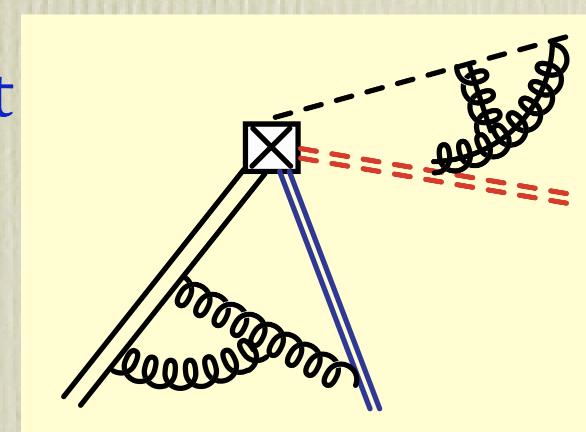
1) Match QCD onto SCET:



2) Decouple Collinear and Soft:



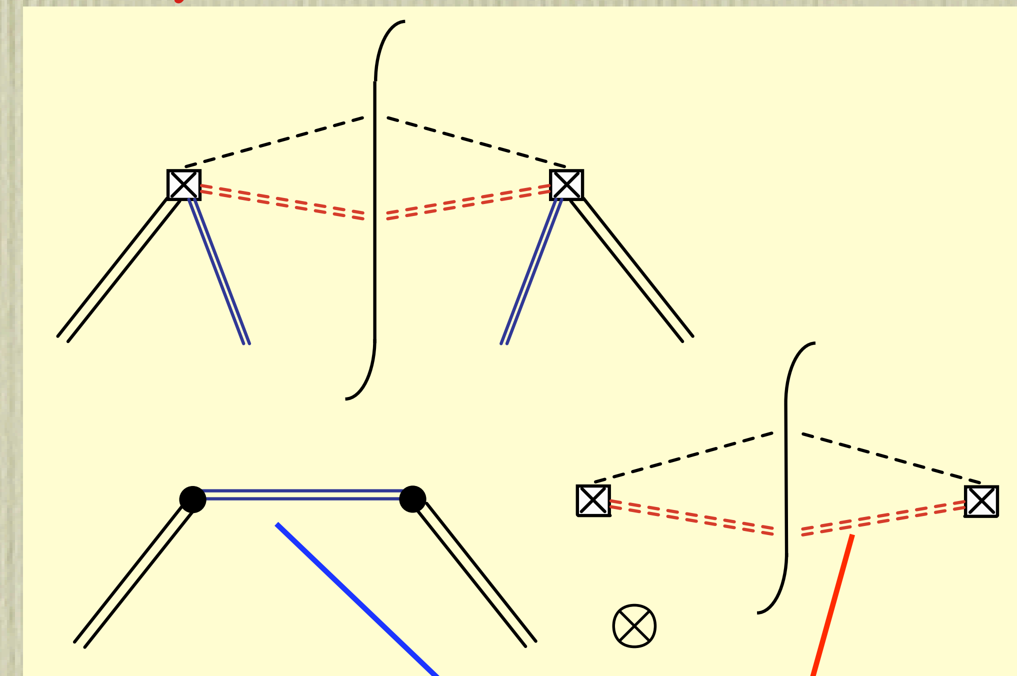
- Heavy/Soft do not interact w. Collinear



$B \to X_s\gamma$ in SCET

3) Factor Decay Rate:

$$\frac{d\Gamma}{dz} =$$



$$=$$

$$\frac{d\Gamma}{dz} = C(M,\mu)\int_z^1 d\eta\, S(\eta,\mu) J(\eta - z,\mu)$$

shape function

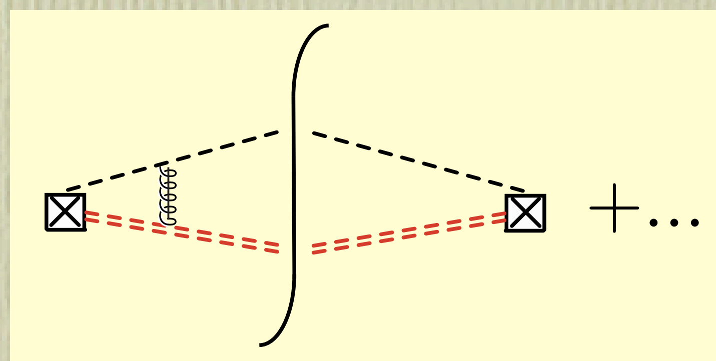
4) O.P.E.: Integrate out $J(\eta - z,\mu)$ at the scale $\sqrt{M(1-z)} \sim \sqrt{M\Lambda_{\mathrm{QCD}}}$

- Perturbatively in expansion in $\alpha_s(\sqrt{M(1-z)})$

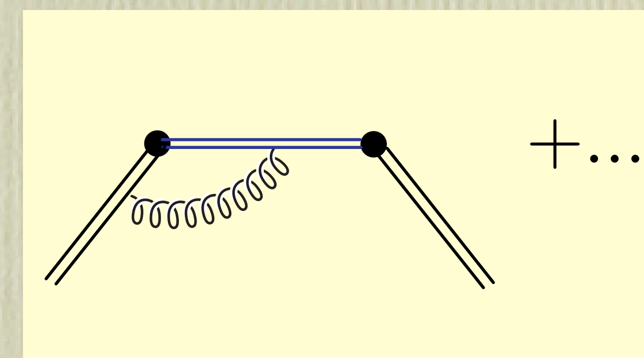
$B \to X_s\gamma$ in SCET

5) Sum Large Logarithms

- Anomalous dimension:



$\to \gamma_C$



$\to \gamma_S$

- Run $J(\eta - z, \mu)$ from M to $\sqrt{M(1-z)} \sim \sqrt{M\Lambda_{\rm QCD}}$
- Run $S(\eta, \mu)$ from M to $M(1-z) \sim \Lambda_{\rm QCD}$

Use RGEs

6) Subleading corrections?

That's being worked on!

What we Learned so Far...

- SCET: EFT of collinear d.o.f. coupled to soft d.o.f.
  - Powerful gauge symmetries constrain operators
  - Decoupling via field redefinition
- Factorization using algebraic methods
- Sum phase space logs using RGEs
- Systematically incorporate power corrections in λ
- Showed how previous results on $B \to X_s \gamma$ near the endpoint are simply reproduced from the unified picture of SCET

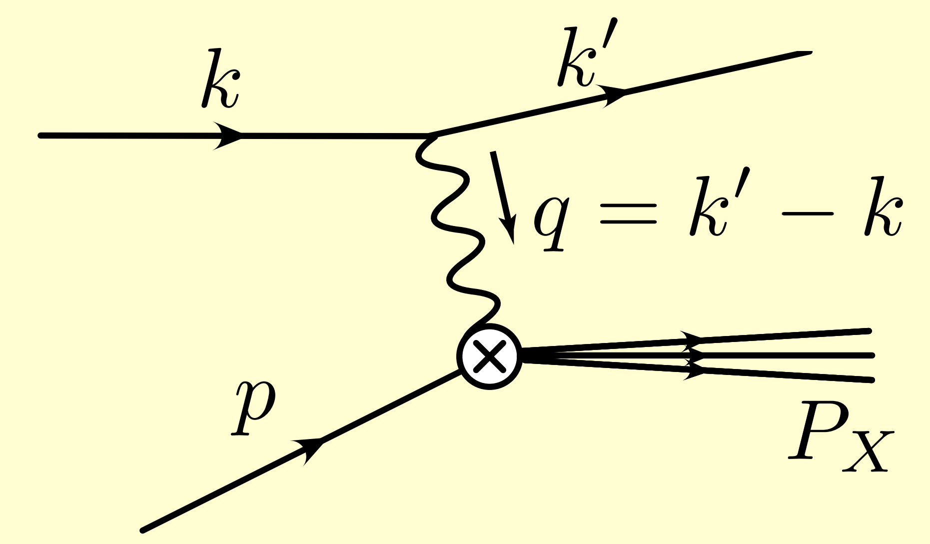
Some More Applications

- Deep Inelastic Scattering
- J/ψ Production at Belle & Babar

Hard Scattering Factorization

- **C. Bauer, SF, D. Pirjol, I. Rothstein, I. Stewart, Phys. Rev. D66: 014017, 2002**
- Derived factored forms for
  - Exclusive: $\pi - \gamma$ form factor $(\gamma\gamma^* \to \pi^0)$
    light meson form factor $(\gamma^* M \to M)$
  - Inclusive: deep inelastic scattering $(e^- p \to e^- X)$
    Drell-Yan $(p\bar{p} \to X\ell^+\ell^-)$
    deeply virtual Compton scattering $(\gamma^* p \to \gamma^{(*)} p)$

Factorization in DIS



- Kinematics: Breit frame

$$q^\mu = Q(\bar{n}^\mu - n^\mu)/2 \qquad \textbf{with} \qquad q^2 = -\frac{\bar{n}\cdot n}{2} Q^2 = -Q^2$$

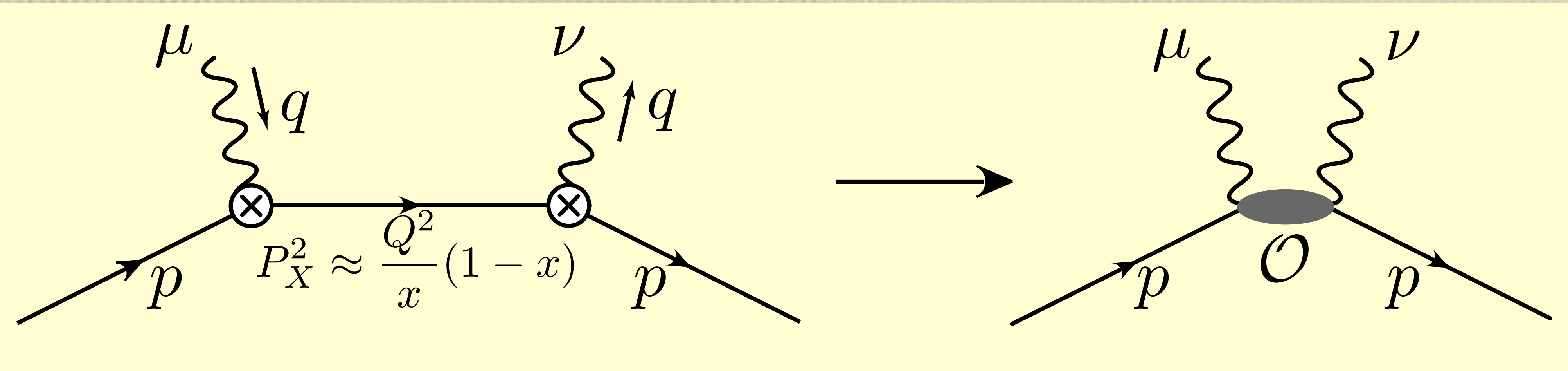
$$x = \frac{Q^2}{2p\cdot q}$$

$$p^\mu = n^\mu \frac{Q}{2x} + \bar{n}^\mu x \frac{m_p^2}{2Q} + \mathcal{O}\left(\frac{m_p^2}{Q^2}\right)$$

$$P_X^\mu = p^\mu + q^\mu \qquad \textbf{with} \qquad \boxed{P_X^2 = \frac{Q^2}{x}(1-x) + m_p^2}$$

Factorization in DIS

- OPE: integrate out final state below the scale Q^2 by matching onto SCET



$$T_{\mu\nu}^{\rm eff} \sim \int d\omega_1\, d\omega_2\, C_{\mu\nu}(\omega_1,\omega_2)\, \mathcal{O}(\omega_1,\omega_2)$$

depends on large light-cone momentum in hard scattering

- SCET operators $\mathcal{O}(\omega_1,\omega_2) = \left[\bar\chi_{n,\omega_1} \frac{\bar{n}\!\!\!/}{2} \chi_{n,\omega_2}\right]$

$$\chi_{n,\omega} = [W^\dagger \xi_n]_\omega$$

- Fix $C_{\mu\nu}(\omega_1,\omega_2)$ by forcing $T_{\mu\nu} = T_{\mu\nu}^{\rm eff}$

Factorization in DIS

- Decouple soft from Collinear $\chi_{n,\omega} \to Y \chi^{(0)}_{n,\omega}$

$$\left[\bar{\chi}_{n,\omega_1} \frac{\bar{n}\!\!\!/}{2} \chi_{n,\omega_2}\right] \to \left[\bar{\chi}^{(0)}_{n,\omega_1} \frac{\bar{n}\!\!\!/}{2} Y^\dagger Y \chi^{(0)}_{n,\omega_2}\right]$$

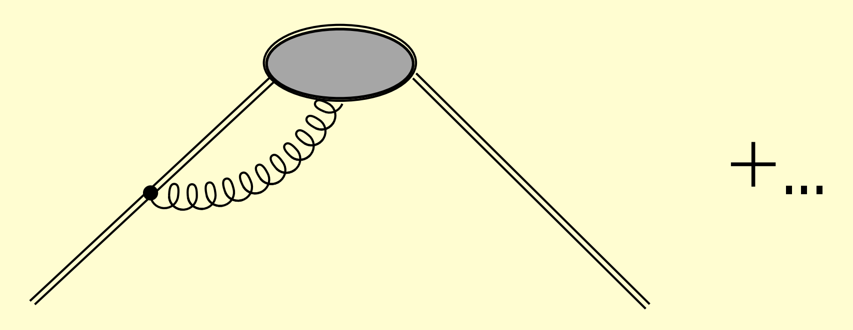
$Y^\dagger Y \to 1$

KLN Theorem

- Parton distributions in SCET

$$\frac{1}{4}\sum_{\text{spin}} \langle p_n | \bar{\chi}_{n,\omega}\, \bar{n}\!\!\!/\, \chi_{n,\omega'} | p_n \rangle = \int_0^1 d\xi\, \delta(\omega_-)\delta\left(\frac{\omega_+}{2\bar{n}\cdot p} - \xi\right) f_{i/p}(\xi)$$

- Operator running



$$\longrightarrow \gamma$$

RGE $$\mu \frac{d}{d\mu} \mathcal{O}(\omega_1, \omega_2; \mu) = \int dx dy\, \gamma(\omega_1, \omega_2, x, y; \mu) \mathcal{O}(x, y; \mu)$$

Different momentum constraints give different running:

$\omega_1 = \omega_2 \to$ DGLAP or $\omega_1 + \omega_2 = \text{Const.} \to$ BL

Factorization in DIS

- Factored form $d\sigma \sim \int \frac{d\xi}{\xi} H\left(\frac{\xi}{x};\mu\right) f_{i/p}(\xi;\mu)$

  $\sim Q$ $\quad\quad\quad\quad$ $\sim \Lambda_{\mathrm{QCD}}$

DIS in the Endpoint Region: $x \to 1$

A. Manohar, Phys. Rev. D68: 114019, 2003

$$P_X^2 \approx Q^2(1-x)$$

- For $1-x \sim \frac{\Lambda_{\mathrm{QCD}}}{Q}$ the final state is collinear: $P_X^2 \sim Q\Lambda_{\mathrm{QCD}}$
  - Sensitive to $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ motion of the quark in proton

New non-perturbative function

$$d\sigma \sim \underbrace{H(Q;\mu)}_{\sim Q} \int \frac{d\xi}{\xi} \underbrace{J(\frac{x}{\xi};\mu)}_{\sim \sqrt{Q^2(1-x)}} \int \frac{d\omega}{\omega} f_{i/p}(\frac{\xi}{\omega};\mu) \underbrace{S(\omega;\mu)}_{\sim \Lambda_{\mathrm{QCD}}}$$

- Sum logs of various scales using RGEs

DIS with Massive Quarks

SF: work in progress

- How to treat heavy-quarks in DIS?
- Are they partons?

Is the β-function calculated using the heavy-quark as an active flavor?

Is there a heavy-quark pdf?

- Much work on the subject: still controversial

SCET has something to say!

Add massive collinear particle $p_h^2 = M^2$ to SCET

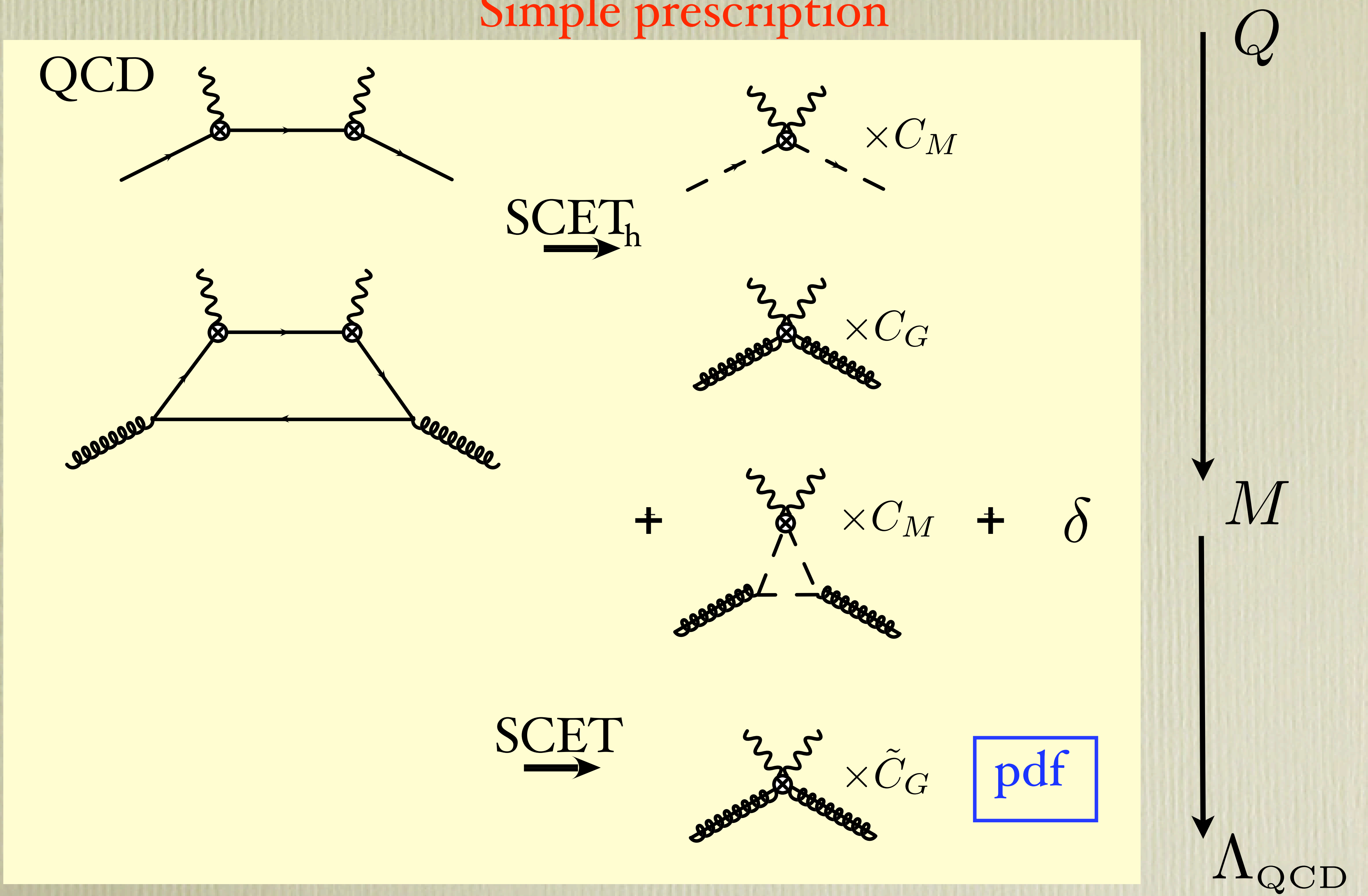
New expansion parameter: $p_h^2 \sim Q^2 \lambda_h^2 \to \lambda_h \sim \dfrac{M}{Q}$

Valid between the scales Q and M.

Below M integrate out heavy by matching onto massless SCET

DIS with Massive Quarks

Simple prescription



Summary of Hard Scattering Factorization

- DIS as a particular example
  - Factorization from form of SCET operators
  - General running from RGEs
    - Specific kinematics give DGLAP or BL
  - KLN cancellation of soft through decoupling
  - Same framework: DIS at endpoint
    - Soft do not cancel: new non-perturbative function
  - Systematic approach for massive quarks
- Systematically include power corrections in powers of λ
  - Important for Drell-Yan

One More Application

- J/ψ Production at Belle & Babar

SF, A. Leibovich, T. Mehen, Phys. Rev. D68:094011, 2003

J/ψ Production at Belle & Babar

$$e^+e^- \to J/\psi + X \; (\sqrt{s} = 10.6\,\mathrm{GeV})$$

Angular distribution

$$\frac{d\sigma}{dp\; dcos\theta} = S(p)(1 + A(p)cos^2\theta)$$

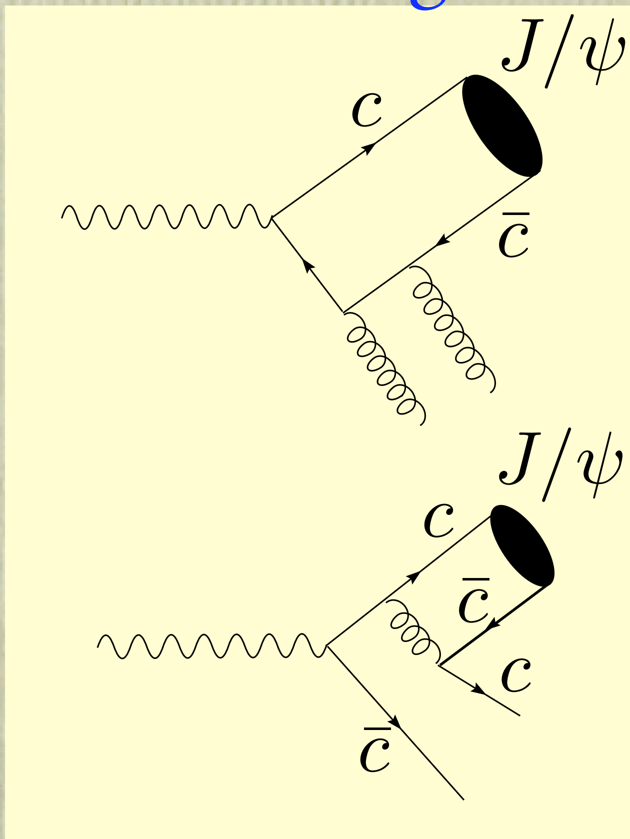
$$A(p)$$

	$\sigma_{tot}(\mathrm{pb})$	$p \lesssim 3.5$ GeV	$p \gtrsim 3.5$ GeV
Babar	$2.52 \pm 0.21 \pm 0.21$	0.05 ± 0.22	1.5 ± 0.6
Belle	$1.47 \pm 0.10 \pm 0.13$	0.7 ± 0.3	$1.1^{+0.4}_{-0.3}$

Belle $$\frac{\sigma(e^+e^- \to J/\psi c\bar{c})}{\sigma(e^+e^- \to J/\psi X)} = 0.59^{+0.15}_{-0.13} \pm 0.12$$

NRQCD

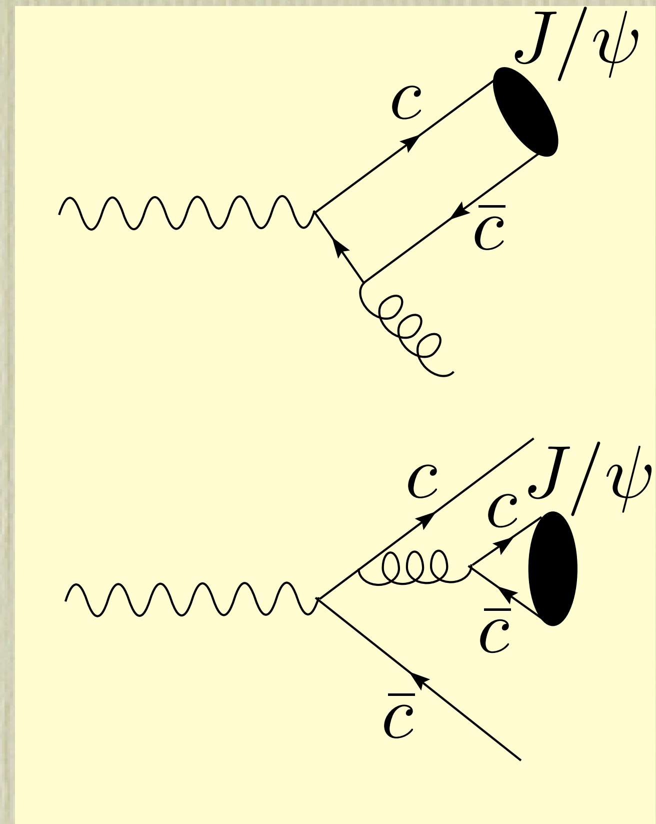
Color Singlet



$\sigma = 0.73$ pb

$\sigma = 0.20$ pb

Color Octet



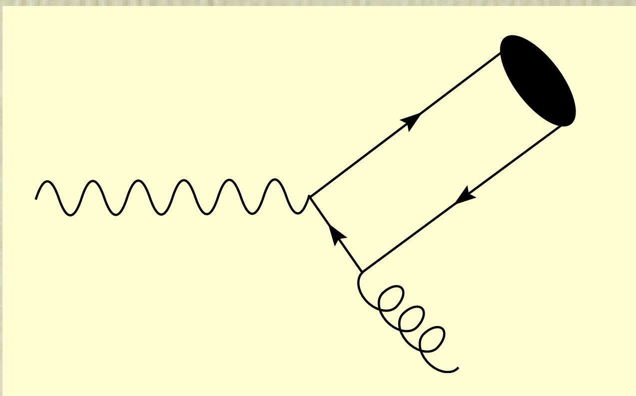
$\sigma = 0.79$ pb

$\sigma = 0.08$ pb

$$\sigma^{(1)}_{\mathrm{tot}} = 0.93 \text{ pb} + \sigma^{(8)}_{\mathrm{tot}} = 0.87 \text{ pb} \rightarrow \sigma_{\mathrm{tot}} = 1.8 \text{ pb}$$

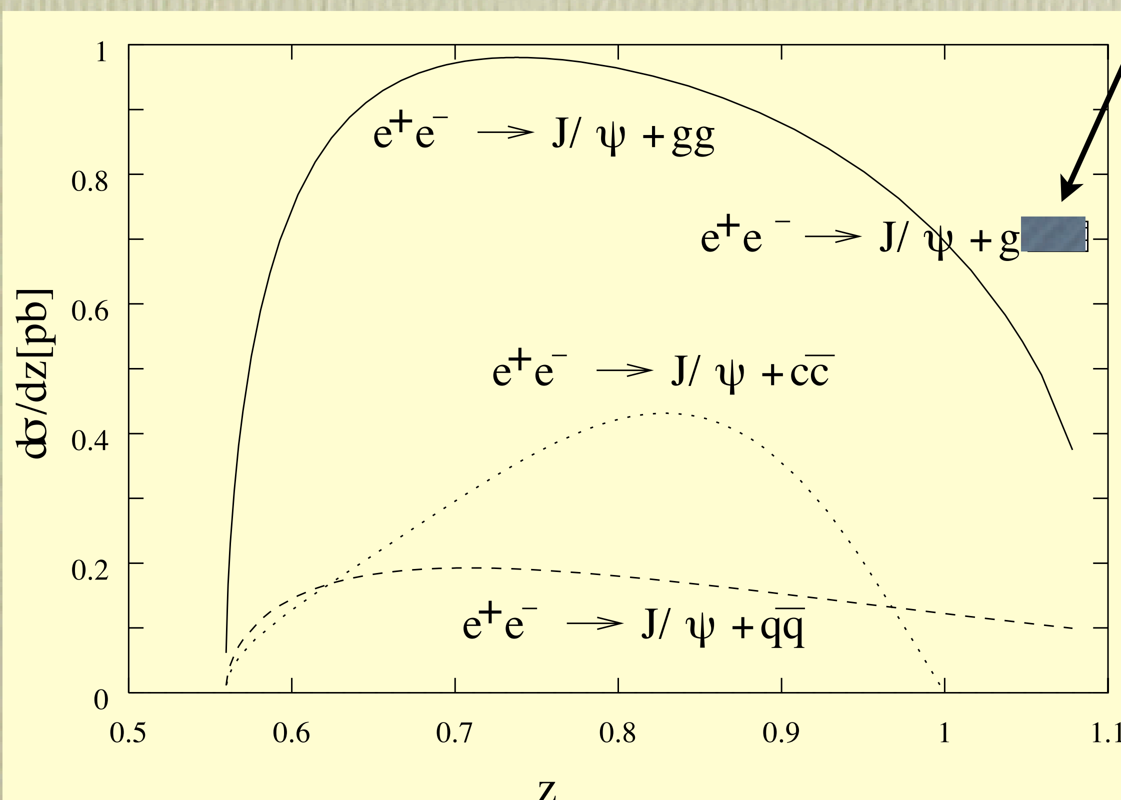
$$\frac{\sigma(e^+e^- \rightarrow J/\psi \; c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi \; X)} = 0.1$$

Differential Distribution

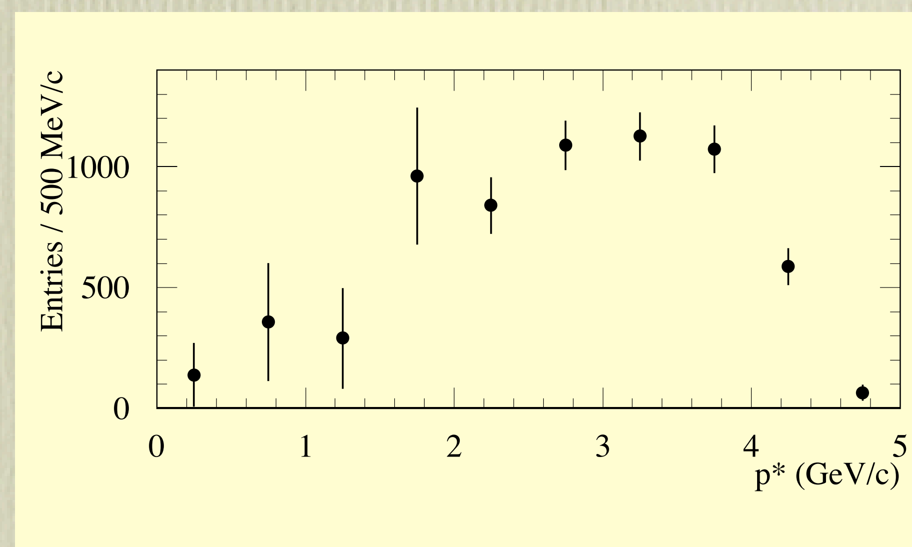


$$\rightarrow d\sigma \propto \delta(1 - z_p) \qquad z_p = p/p_{\max}$$

Theory

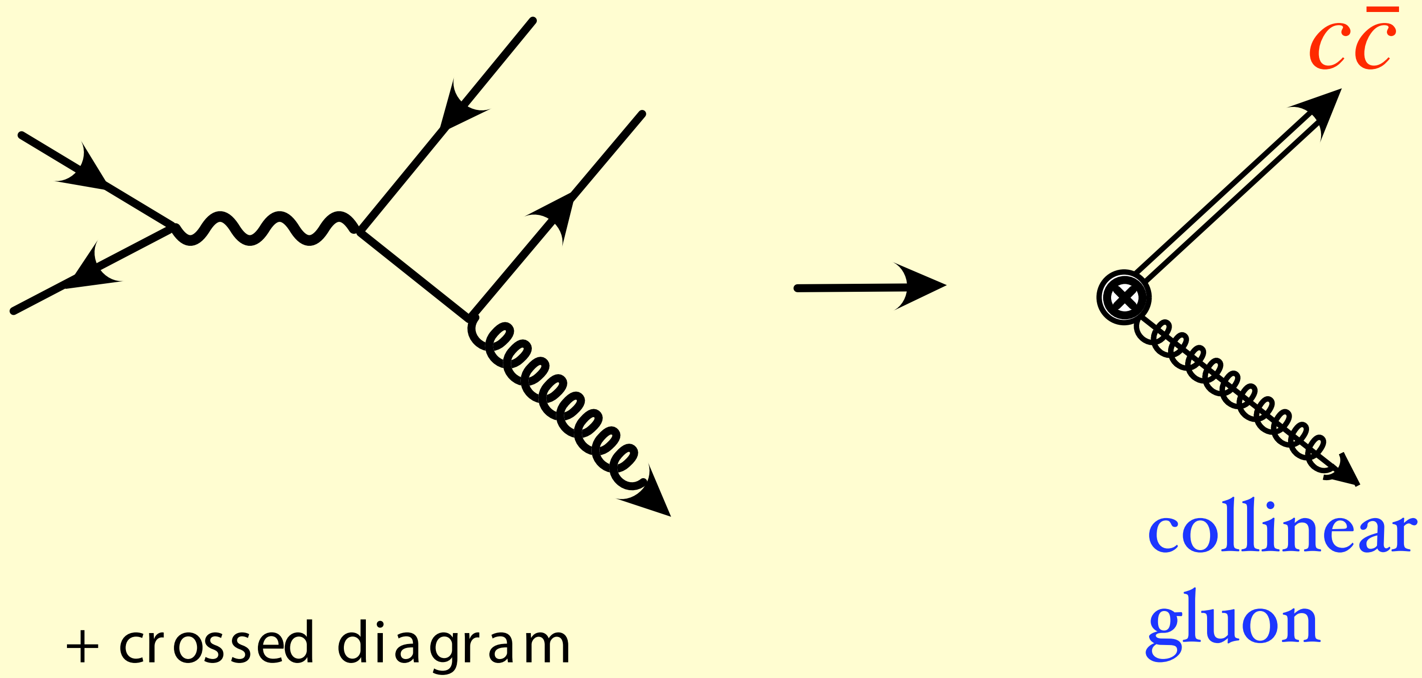


Babar



SCET & NRQCD

- In the Endpoint region:
  - use SCET for the fast & soft d.o.f.
  - use NRQCD for the heavy quark-antiquark



Factorization

- New factorization formula in the endpoint region:

(Similar to $B \to X_s \gamma$)

Nonperturbative shape function

$$\frac{d\sigma}{dz} \propto \int_z^1 d\xi\, S(\xi;\mu) J(\xi - z;\mu)$$

Jet function

Jet function: perturbatively calculable in $\alpha_s\left(\sqrt{\frac{s}{m_c}\Lambda_{QCD}}\right)$

Shape function is universal

Used a simple model with 2 parameters: require moments to scale appropriately

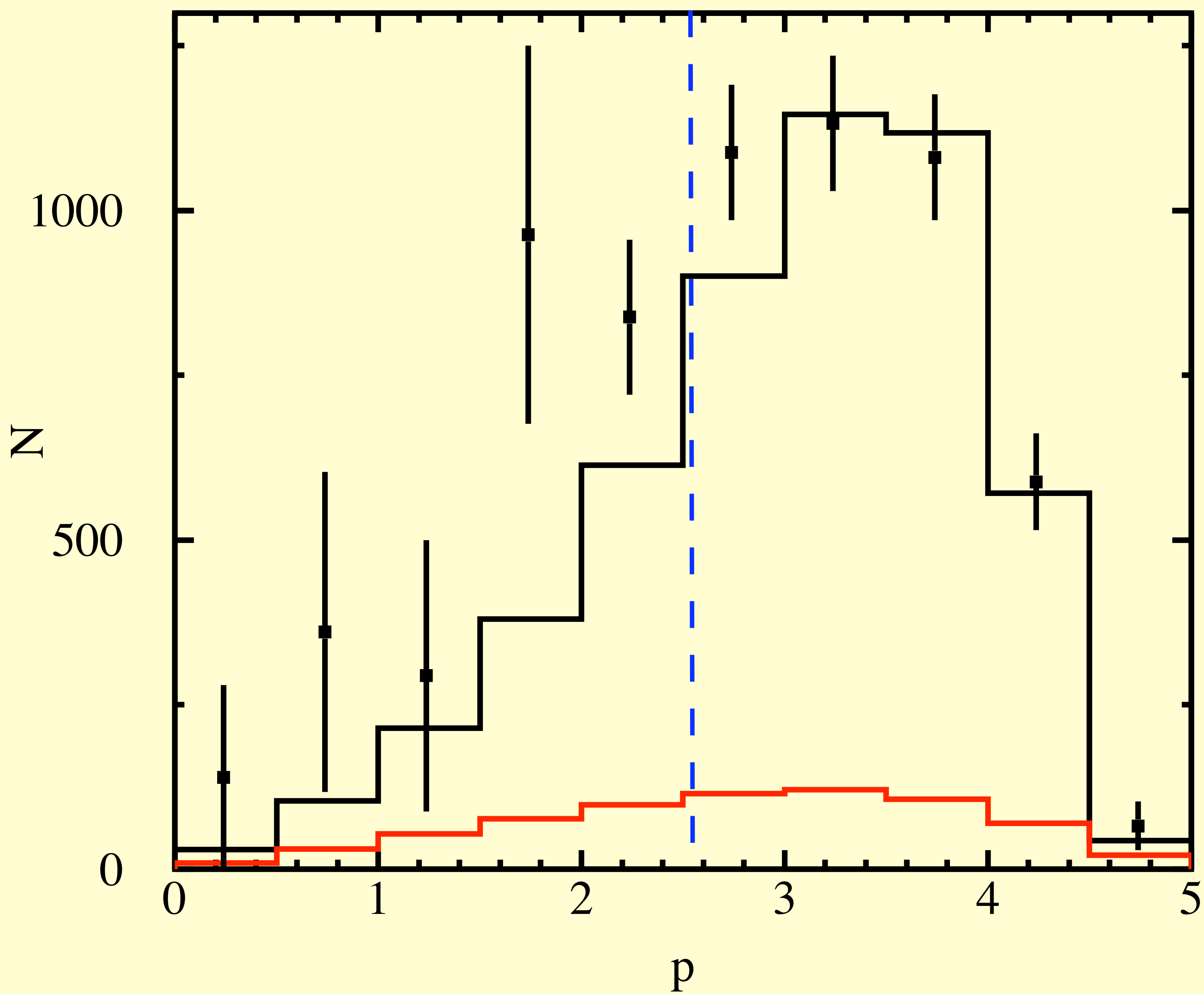
Overall normalization includes color-octet matrix element

Not well determined

Sum logs using RGEs

Comparison to Babar Data

B. Aubert *et al.* Phys. Rev. Lett. 87: 162002 (2001)



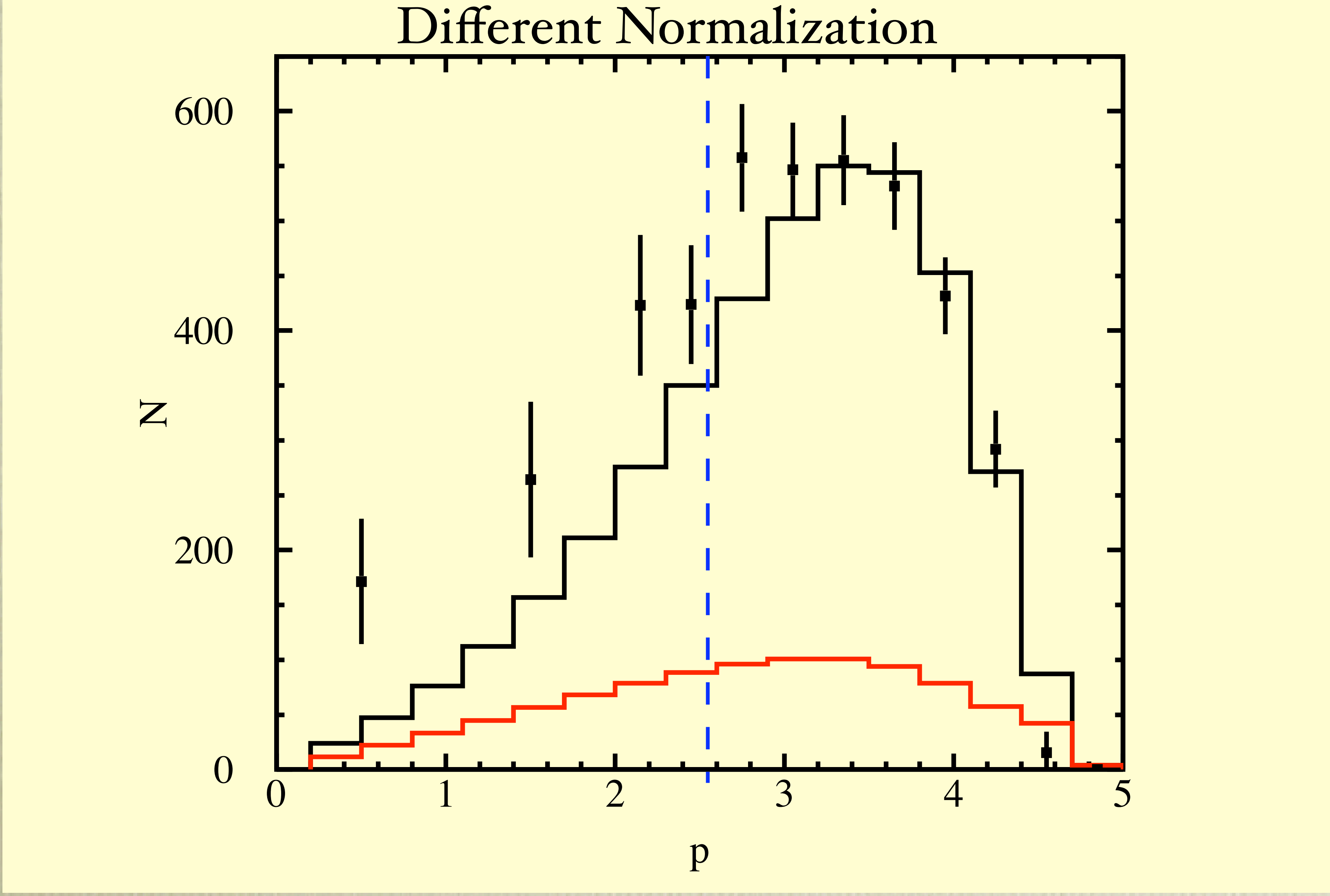
Endpoint

$z \gtrsim 0.7$

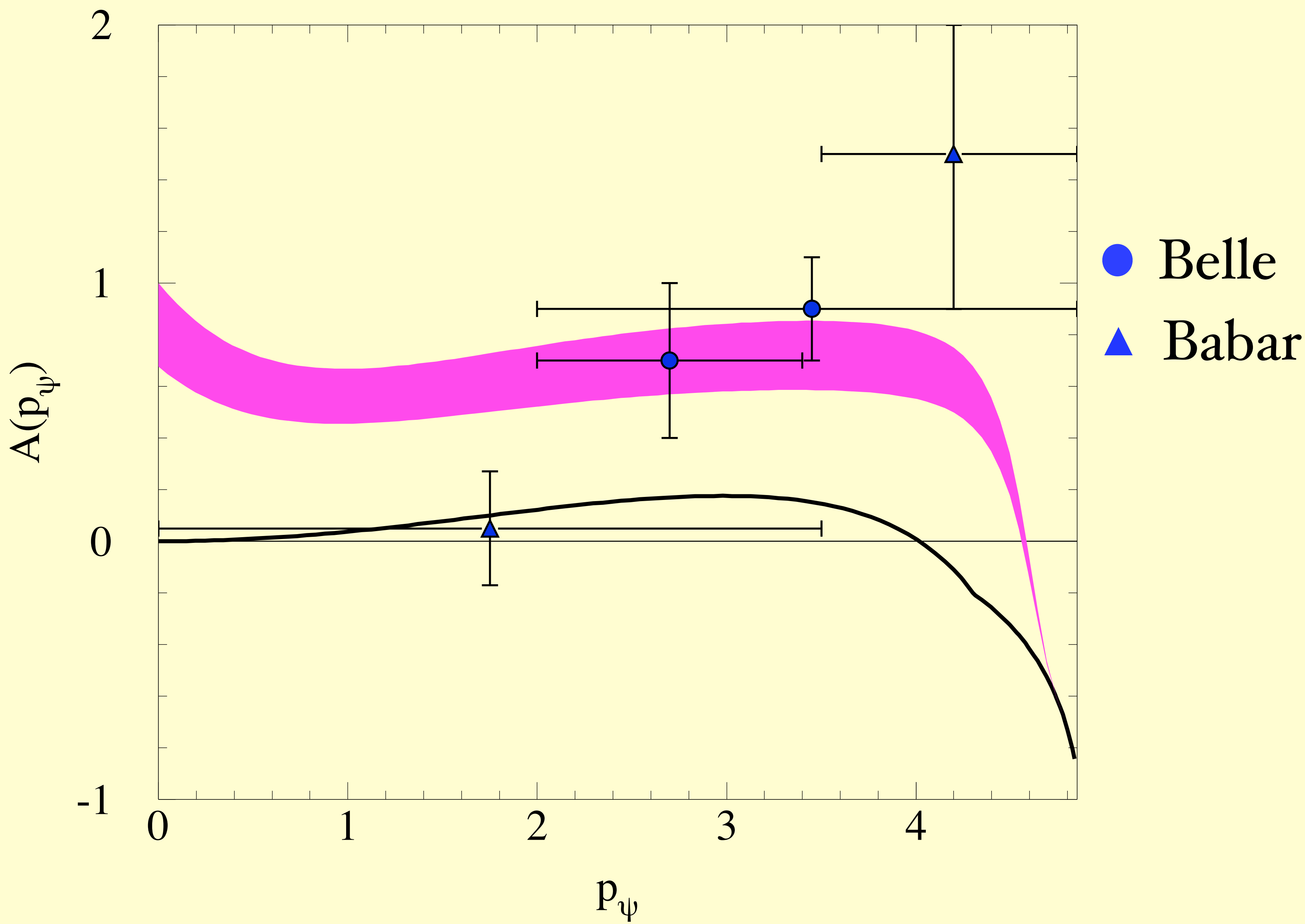
$p \gtrsim 2.57$ GeV

Comparison to Belle Data

K. Abe *et al.* Phys. Rev. Lett. 88: 052001 (2002)



Angular Distribution



Summary of $e^+e^- \to J/\psi + X$

- The color-octet contribution is needed to explain σ_{tot}
  - Contributes mainly in endpoint region
- Need to incorporate collinear physics to get a sensible prediction for $d\sigma/dp$
- Prediction for $d\sigma/dp$ and angular distribution consistent with data
- Model for shape function & arbitrary normalization
  - Extract from other process*
- Charm fraction $\dfrac{\sigma(e^+e^- \to J/\psi c\bar{c})}{\sigma(e^+e^- \to J/\psi X)}$ still a mystery
  - Does factorization breakdown? *

* In progress

The Tip of the Iceberg

- In this talk
  - $B \to X_s\gamma$ in endpoint region
  - DIS
  - $e^+e^- \to J/\psi + X$ @ Belle & Babar
- Some More
  - $B \to D\pi$, $B \to \pi\ell\nu$, $B \to \pi\pi$, $B \to \gamma\ell\nu$, $B \to X_u\ell\nu$, $\Upsilon \to \gamma X$, $\gamma\gamma^* \to \pi$, $\gamma^* M \to M$, $\gamma^* p \to \gamma^{(*)} p$, $p\bar{p} \to X\ell^+\ell^-$, Jet distributions in e^+e^- annihilation, Power corrections
- Visions of the Future
  - Massive quarks in DIS
  - Jets in e^+e^- annihilation
  - $\gamma^{(*)}p \to J/\psi + X$
  - Apply SCET to multi-scale process in $p\bar{p}$ collisions
  - Electroweak Sudakov logs

Summary & Conclusions

- Flavor of Soft Collinear Effective Theory
  - Theory of light-like particles interacting with a soft background
  - Derive factorization
  - Sum logarithms
  - Systematically treat power corrections
- Scope of applications is large
  - Examples: B decays to DIS
- A very active field
- Only scratched the surface: so much left to do...